Θέμα 4ο

Α. $\lambda'(t) = 2$ cm/sec και $\lambda(t) = \varepsilon\varphi(\theta(t))$

$\lambda(t) = 2t + c$ για $t=0$ έχουμε $y = \lambda x \xrightarrow{M(2,1)} \lambda(0) = \frac{1}{2}$

Άρα $\lambda(0) = \frac{1}{2} \Rightarrow c = \frac{1}{2}$ οπότε $\lambda(t) = 2t + \frac{1}{2}$

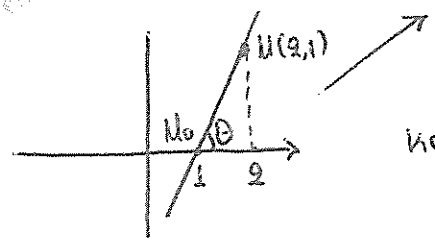
Β. $\lambda(t) = \varepsilon\varphi(\theta(t))$

$\lambda(0) = \varepsilon\varphi(\theta(0)) \Rightarrow \varepsilon\varphi(\theta(0)) = \frac{1}{2}$

Άρα $\varepsilon\varphi(\theta(t)) = 2t + \frac{1}{2}$ τότε $(\varepsilon\varphi(\theta(t)))' = (2t + \frac{1}{2})'$

$$\frac{1}{\sigma\upsilon\nu^2\theta(t)} \cdot \theta'(t) = 2 \Leftrightarrow \theta'(t) = 2\sigma\upsilon\nu^2\theta(t) \ \frac{\text{rad}}{\text{sec}}$$

Γ1. $M_0(1,0)$ οπότε $\theta_0 = 45°$



$\lambda_0 = \varepsilon\varphi\theta_0 = \varepsilon\varphi 45° = 1$

$\theta'(t_0) = 2\sigma\upsilon\nu^2 45 = 2 \cdot (\frac{\sqrt{2}}{2})^2 = 1$ rad/sec

και $\lambda_0 = 2t_0 + \frac{1}{2}$

$1 = 2t_0 + \frac{1}{2} \Rightarrow 2 = 4t_0 + 1 \Rightarrow t_0 = \frac{1}{4}$ sec

Γ2. $t_0 = \frac{1}{4}$ sec $M(2,1)$

ε: $y - y_0 = \lambda(x - x_0) \Rightarrow y - 1 = \lambda(x-2) \Leftrightarrow y - 1 = \lambda x - 2\lambda \Leftrightarrow y = \lambda x - 2\lambda + 1$

Για $y=0$: $\lambda x - 2\lambda + 1 = 0 \Leftrightarrow \lambda x = 2\lambda - 1 \Leftrightarrow x = \frac{2\lambda - 1}{\lambda}$

$A(\frac{2\lambda-1}{\lambda}, 0)$

$$E = \frac{1}{2} B \cdot \upsilon = \frac{1}{2} \cdot (2 - \frac{2\lambda-1}{\lambda}) \cdot 1 = \frac{1}{2} \cdot \frac{2\lambda - 2\lambda + 1}{\lambda} = \frac{1}{2\lambda}$$

$E(t) = \frac{1}{2\lambda(t)}$ Άρα $E'(t) = -\frac{1}{2} \cdot \frac{1}{\lambda^2(t)} \cdot \lambda'(t) \Rightarrow E'(t) = -\frac{\lambda'(t)}{2\lambda^2(t)}$

$$E(t_0) = -\frac{\lambda'(t_0)}{2\lambda^2(t_0)} = -\frac{2}{2 \cdot 1} = -1 \ \text{cm}^2/\text{sec}$$

$\lambda(t_0) = 1$
$\lambda'(t_0) = 2$