Θέμα 4ο

Α. $f'(x) = (x)' + (e^{-mx})' = 1 - e^{-mx}\cdot m$

$f'(x)=0 \Leftrightarrow 1 - me^{-mx} = 0 \Leftrightarrow e^{-mx} = \frac{1}{m} \Leftrightarrow \ln e^{-mx} = \ln\frac{1}{m} \Leftrightarrow -mx = -\ln m \Leftrightarrow x = \frac{\ln m}{m}$

x	$-\infty$	$\ln m/m$	$+\infty$
f'	$-$	0	$+$
f	$\searrow$	Ο.Ε.	$\nearrow$

Ο.Ε. το $g(m) = f\left(\frac{\ln m}{m}\right) = \frac{\ln m}{m} + e^{-m\frac{\ln m}{m}} = \frac{\ln m}{m} + m^{-1} = \frac{\ln m + 1}{m}$

Β. $g(m) = \frac{1+\ln m}{m}$, $g'(m) = -\frac{\ln m}{m^2}$

$g'(m) = 0 \Leftrightarrow m = 1$

m	$-\infty$	0		1	$+\infty$
g'			$+$	0	$-$
g			$\nearrow$	Ο.Μ.	$\searrow$

$g(1) = f(c) = 1$

Γ1. $ημx + e^{-ημx} \le 1 \Leftrightarrow f(ημx) \le 1$
όμως $f(x) \ge 1$

$\Rightarrow ημx = 0 \Leftrightarrow ημx = ημ0 \Leftrightarrow \begin{cases} x = 2κπ \\ x = 2κπ + π \end{cases}$ $κ \in \mathbb{Z}$

Γ2. $f(x) = x + e^{-x}$, $f'(x) = 1 - e^{-x}$

ε: $y - f(x_0) = f'(x_0)(x - x_0) \overset{(0,0)}{\Longleftrightarrow} -f(x_0) = -x_0\cdot f'(x_0) \Leftrightarrow x_0 + e^{-x_0} = x_0(1 - e^{-x_0})$

$\Leftrightarrow \cancel{x_0} + e^{-x_0} = \cancel{x_0} - x_0e^{-x_0} \Leftrightarrow e^{-x_0}(1 + x_0) = 0 \Leftrightarrow x_0 = -1$

$f(-1) = e - 1$
$f'(-1) = 1 - e$

Άρα ε: $y - (e-1) = (1-e)(x+1) \Leftrightarrow y = (1-e)x$

Γ3. $A(0,2)$

$y - f(x_0) = f'(x_0)(x - x_0) \Leftrightarrow 2 - f(x_0) = f'(x_0)(0 - x_0) \Leftrightarrow$

$\Leftrightarrow 2 - f(x_0) = -x_0 f'(x_0) \Leftrightarrow 2 - f(x_0) + x_0 f'(x_0) = 0$

Θεωρούμε $h(x) = 2 - f(x) + x f'(x)$

$h'(x) = -\cancel{f'(x)} + \cancel{f'(x)} + x f''(x)$

$h'(x) = x \cdot f''(x)$

$f''(x) = e^{-x} > 0$

x	$-\infty$	0	$+\infty$
h'	$-$	0	$+$
h	$\searrow$	Ο.Ε.	$\nearrow$

$h(x) \ge h(0) \Leftrightarrow h(x) \ge 1 \Rightarrow h(x) > 0$

οπότε $h(x) \neq 0$

Άρα δεν υπάρχει εφαπτομένη που να διέρχεται από το $A(0,2)$