Θέμα 34

Α. $\varepsilon: y - f(x_0) = f'(x_0)(x - x_0)$ $\quad M_0(x_0, f(x_0))$, $M(x_0, 0)$

$\Sigma(\frac{x_0}{2}, 0)$ $\qquad O\Sigma = \Sigma M$

Άρα $-f(x_0) = f'(x_0)\left(\frac{x_0}{2} - x_0\right)$

$-f(x_0) = -\frac{x_0}{2} \cdot f'(x_0) \Leftrightarrow 2f(x_0) = f'(x_0) \cdot x_0$

Β. $F(x) = \frac{f(x)}{x^2}$

$F'(x) = \frac{f'(x) \cdot x^2 - f(x) \cdot 2x}{x^4} = \frac{f'(x) \cdot x - 2f(x)}{x^3} = \frac{0}{x^3} = 0$ $\qquad$ Άρα $F(x) = c$

Γ. $\left.\begin{array}{l} F(x) = c \Leftrightarrow \frac{f(x)}{x^2} = c \Leftrightarrow f(x) = c \cdot x^2 \quad \text{για } x > 0 \\ \text{Για } x = 0 \text{ αφού } f \text{ συνεχής } : f(0) = \lim_{x \to 0} f(x) = 0 \end{array}\right\} \quad f(x) = cx^2 , \; x \geq 0$

Δ1. $f(x) \geq 2x - 2 + c \Leftrightarrow f(x) - 2x + 2 - c \geq 0$ $\qquad f'(x) = 2cx$

Θεωρώ $h(x) = f(x) - 2x + 2 - c$ $\quad$ με $h'(x) = f'(x) - 2$

$h(1) = f(1) - \cancel{2} + \cancel{2} - c = c - c = 0$

Άρα $h(x) \geq h(1)$ οπότε η παρουσιάζει ελάχιστο στο $x = 1$

οπότε $h'(1) = 0 \Leftrightarrow f'(1) - 2 = 0 \Leftrightarrow f'(1) = 2 \Leftrightarrow 2c = 2 \Leftrightarrow c = 1$

Άρα $f(x) = x^2$, $x \geq 0$

Δ2. $\lim_{x \to r} \frac{e^{2x} - 1}{f(x)} = +\infty$

Αν $r = 0$: $\lim_{x \to 0} \frac{e^{2x} - 1}{f(x)} \overset{\frac{0}{0}}{\underset{DLH}{=}} \lim_{x \to 0} \frac{2e^{2x}}{f'(x)} = \lim_{x \to 0} \frac{\cancel{2}e^{2x}}{\cancel{2}x} = +\infty$

Αν $r > 0$: τότε $f(r) = r^2 > 0$

$\lim_{x \to r} \frac{e^{2x} - 1}{f(x)} = \frac{e^{2r} - 1}{f(r)} \in \mathbb{R}$

Άρα $r = 0$