Θέμα 2ο

Α1. $f(x) = x^6 - 2x^3 + 2 \ , \ x \in \mathbb{R}$

$f'(x) = 6x^5 - 6x^2$

$f'(x) = 0 \Leftrightarrow 6x^2(x^3-1) = 0 \Leftrightarrow x = 0$ ή $x = 1$

x	$-\infty$		0		1		$+\infty$
f'		$-$	0	$-$	0	$+$	
f		$\searrow$		$\searrow$	Ο.Ε.	$\nearrow$	

$f(1) = 1$

Α2. $g(x) = -x^4 + 2x^2 \ , \ x \in \mathbb{R}$

$g'(x) = -4x^3 + 4x$

$g'(x) = 0 \Leftrightarrow -4x(x^2-1) = 0 \Leftrightarrow x = 0$ ή $x = \pm 1$

x	$-\infty$		-1		0		1		$+\infty$
$-4x$		$+$		$+$	0	$-$		$-$	
x^2-1		$+$	0	$-$		$-$	0	$+$	
g'		$+$	0	$-$	0	$+$	0	$-$	
g		$\nearrow$	Τ.Μ	$\searrow$	Τ.Ε	$\nearrow$	Τ.Μ	$\searrow$	

$f(\Delta_1) = \left(\lim\limits_{x\to-\infty} g(x), g(-1)\right) = (-\infty, 1)$

$f(\Delta_2) = [g(0), g(-1)] = [0, 1]$

$f(\Delta_3) = (g(0), g(1)] = (0, 1)$

$f(\Delta_4) = \left(\lim\limits_{x\to+\infty} g(x), g(1)\right) = (-\infty, 1)$

$g(\mathbb{R}) = (-\infty, 1]$

Άρα g ο.μ. το 1

Β1. $x^6 + x^4 - 2x^3 - 2x^2 + 2 = 0 \Leftrightarrow (x^6 - 2x^3 + 2) - (-x^4 + 2x^2) = 0 \Leftrightarrow f(x) = g(x)$

Έχουμε $f(x) \geq 1$ και $g(x) \leq 1$ Άρα $x = 1$

Β2. $f(r) = ημ r$

Έστω ότι υπάρχει αριθμός r

Ξέρουμε $ημ r \leq 1$

$f(r) \geq 1 \rightarrow f(r) = 1 \Rightarrow r = 1$ τότε $ημ1 = 1$ Άτοπο

Β3. $ε_f: \ y - f(1) = f'(1)(x-1) \Leftrightarrow y - 1 = 0 \Leftrightarrow y = 1$

$ε_g: \ y - g(1) = g'(1)(x-1) \Leftrightarrow y - 1 = 0 \Leftrightarrow y = 1$

Γ. $\lim\limits_{x\to+\infty} \dfrac{ημ f(x)}{g(x)} = \lim\limits_{x\to+\infty} \left(\dfrac{ημ f(x)}{-x^4+2x^2} \right)$ $\quad -1 \leq ημ f(x) \leq 1 \Leftrightarrow \dfrac{-1}{-x^4+2x^2} \geq \dfrac{ημ f(x)}{g(x)} \geq \dfrac{1}{-x^4+2x^2}$

$\lim\limits_{x\to+\infty} \dfrac{-1}{-x^2(x^2+2)} = 0$ και $\lim\limits_{x\to+\infty} \dfrac{1}{-x^2(x^2-2)} = 0$

Από κ.π. $\lim\limits_{x\to+\infty} \dfrac{ημ f(x)}{g(x)} = 0$