Θέμα 23

Α1. $f(x) = 2ax - \frac{2x}{x^2+1}$

$\lim_{x\to+\infty} \frac{f(x)}{x} = 2$ και $\lim_{x\to+\infty} (f(x)-2x) = 0$

$\hookrightarrow \lim_{x\to+\infty} \left(2a - \frac{2}{x^2+1}\right) = 2 \Leftrightarrow 2a = 2 \Rightarrow a = 1$

Α2. $\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty}\left(2x - \frac{2x}{x^2+1}\right) = \lim_{x\to-\infty} \frac{2x^3+2x-2x}{x^2+1} = \lim_{x\to-\infty} \frac{2x^{\cancel{3}}}{\cancel{x^2}} = -\infty$

$\lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty}\left(2 - \frac{2}{x^2+1}\right) = 2$

$\lim_{x\to-\infty} (f(x)-2x) = \lim_{x\to-\infty}\left(\cancel{2x} - \frac{2x}{x^2+1} - \cancel{2x}\right) = \lim_{x\to-\infty} -\frac{2\cancel{x}}{x^{\cancel{2}}} = 0$

Άρα $y=2x$ πλάγια ασύμπτωτη στο $-\infty$ και $+\infty$

Β1. $f(x) = 2x - \frac{2x}{x^2+1} = \frac{2x^3+2x-2x}{x^2+1} = \frac{2x^3}{x^2+1}$

$f'(x) = \frac{6x^2(x^2+1) - 2x^3 \cdot 2x}{(x^2+1)^2} = \frac{6x^4+6x^2-4x^4}{(x^2+1)^2} = \frac{2x^4+6x^2}{(x^2+1)^2} = \frac{2x^2(x^2+3)}{(x^2+1)^2} \geq 0$

Β2. $f(-x) = 2(-x) - \frac{2(-x)}{(-x)^2+1} = -2x + \frac{2x}{x^2+1} = -f(x)$

Β3. $f''(x) = \left(\frac{2x^4+6x^2}{(x^2+1)^2}\right)' = \frac{(8x^3+12x)(x^2+1)^{\cancel{2}} - (2x^4+6x^2)\cdot 2\cdot \cancel{(x^2+1)}\cdot 2x}{(x^2+1)^{\cancel{4}\,3}} =$

$= \frac{\cancel{8x^5} + 8x^3 + 12x^3 + 12x - \cancel{8x^5} - 24x^3}{(x^2+1)^3} = \frac{-4x^3+12x}{(x^2+1)^3} = \frac{-4x(x^2-3)}{(x^2+1)^3}$

$f''(x) = 0 \Leftrightarrow -4x = 0$ ή $x^2-3=0$

$x = 0$ ή $x = \pm\sqrt{3}$

Σημεία καμπής $M_1(-\sqrt{3}, f(-\sqrt{3}))$

$O(0,0)$

$M_2(\sqrt{3}, f(\sqrt{3}))$

x	$-\infty$		$-\sqrt{3}$		0		$\sqrt{3}$		$+\infty$
$-4x$		+		+	0	−		−	
x^2-3		+	0	−		−	0	+	
f''		+	0	−	0	+	0	−	
f		∪	ΣΚ	∩	ΣΚ	∪	ΣΚ	∩	

Β4. $\lambda_{OM_1} = \frac{f(-\sqrt{3})-0}{-\sqrt{3}-0} = \frac{f(\sqrt{3})}{\sqrt{3}}$ (f περιττή)

$\lambda_{OM_2} = \frac{f(\sqrt{3})-0}{\sqrt{3}-0} = \frac{f(\sqrt{3})}{\sqrt{3}}$

$\Rightarrow \lambda_{OM_1} = \lambda_{OM_2}$

οπότε M_1, O, M_2 συνευθειακά