

## Θέμα 20

A.  $f$  συμμετρική ως προς  $(0,0)$  δηλ περιττή  
 $f(-x) = -f(x)$  για  $x=0$   $f(0) = -f(0) \Rightarrow f(0) = 0$

$$f'(x) = 3x^2 + 1$$

$$f'(1) = 4 \quad \text{Άρα} \quad f(0) = 0 \Rightarrow f'(1) - 4\alpha = 0 \Rightarrow 4 - 4\alpha = 0 \Rightarrow \alpha = 1$$

$$\text{οπότε } f(x) = x^3 + x$$

B.  $f'(x) > 0 \Rightarrow f \uparrow$  άρα  $1-1$  άρα αντιστρέφεται

$$D_{f^{-1}} = f(\mathbb{R}) = (\lim_{x \rightarrow -\infty} f(x), \lim_{x \rightarrow +\infty} f(x)) = (-\infty, +\infty)$$

Π<sub>1</sub>.  $f^{-1}(x) = 0 \Leftrightarrow f(f^{-1}(x)) = f(0) \Leftrightarrow x = f(0) \Leftrightarrow x = 0$

Π<sub>2</sub>.  $f^{-1}(x) < 0 \stackrel{f \uparrow}{\Leftrightarrow} f(f^{-1}(x)) < f(0) \Leftrightarrow x < 0$

$$f^{-1}(x) > 0 \stackrel{f \uparrow}{\Leftrightarrow} f(f^{-1}(x)) > f(0) \Leftrightarrow x > 0$$

Δ.  $(f^{-1}(1))^3 + f^{-1}(1) = f(f^{-1}(1)) = 1$

E.  $(4^x + 2^{x+1})^3 + (4^x + 2^{x+1}) = 30 \cdot f^{-1}(2)$

$$f^{-1}(2) = 1$$

$$f(4^x + 2^{x+1}) = 30$$

$$f(4^x + 2^{x+1}) = f(3) \stackrel{f^{-1}}{\Rightarrow}$$

$$4^x + 2^{x+1} = 3 \Leftrightarrow 2^{2x} + 2 \cdot 2^x - 3 = 0$$

$$\Delta = 4 + 12 = 16$$

$$2^x = \frac{-2 \pm 4}{2} \begin{cases} 2^x = 1 \Leftrightarrow x = 0 \\ 2^x = -3 \text{ ΑΔΥΝΑΤΗ} \end{cases}$$