# Θέμα 08

**Α1.** $f'(x) = \frac{1}{x} + \frac{1}{2\sqrt{x}}$

$f'(x) = (\ln x)' + (\sqrt{x})'$

Άρα $f(x) = \ln x + \sqrt{x} + c$

όμως $f(1) = 1 \Leftrightarrow \cancel{\ln 1}^{0} + \cancel{\sqrt{1}} + c = \cancel{1} \Rightarrow c = 0$

Άρα $f(x) = \ln x + \sqrt{x}$ , $x > 0$

**Α2.** $f(x) = \ln x + \ln e^{\sqrt{x}} = \ln(x \cdot e^{\sqrt{x}})$

$f(x) = (f_1 \circ f_2)(x)$ με $\underset{x>0}{f_1(x) = \ln x}$ και $\underset{x \geq 0}{f_2(x) = x e^{\sqrt{x}}}$

**Β.** $\left.\begin{array}{l} g^2(x) = e^{2x} \neq 0 \Leftrightarrow g(x) \neq 0 \\ g \text{ συνεχής} \end{array}\right\}$ Άρα η $g$ διατηρεί σταθερό πρόσημο

Επειδή $g(e) > 0$ τότε $g(x) > 0$

Άρα $g^2(x) = e^{2x} \Leftrightarrow \underset{\text{Δεκτή}}{g(x) = e^x}$ ή $g(x) = -e^x$ Απορρίπτεται αφού $g(x) > 0$

**Γ.** $h = f \circ g$ $\quad D_{f\circ g} = \begin{cases} x \in D_g \\ g(x) \in D_f \end{cases} = \begin{cases} x \in R \\ e^x > 0 \end{cases} = R$

$h(x) = (f\circ g)(x) = f(g(x)) = f(e^x) = \sqrt{e^x} + \ln e^x = \sqrt{e^x} + x$

**Δ.** $\lim\limits_{x\to 1} \frac{3h(2x) - 3e - 6}{4h'(0)x^3 - 6} = \lim\limits_{x\to 1} \frac{3(h(2x) - e - 2)}{6x^3 - 6} =$

$= \frac{3}{6} \cdot \lim\limits_{x\to 1} \frac{h(2x) - h(2)}{x^3 - 1} = \frac{1}{2} \lim\limits_{x\to 1} \frac{e^x + 2x - e - 2}{x^3 - 1} \overset{DLH}{=}$

$= \frac{1}{2} \lim\limits_{x\to 1} \frac{e^x + 2}{3x^2} = \frac{1}{2} \frac{e+2}{3} = \frac{e+2}{6}$

$h(2x) = \sqrt{e^{2x}} + 2x = e^x + 2x$

$h'(x) = \frac{1}{2\sqrt{e^x}} \cdot e^x + 1$

$h'(0) = \frac{1}{2} + 1 = \frac{3}{2}$

$h(2) = \sqrt{e^2} + 2 = e + 2$