Θέμα 06

A. $f'(x) = \sigma\upsilon\nu x - \eta\mu x$

$f'(x) = (\eta\mu x)' + (\sigma\upsilon\nu x)'$

$f(x) = \eta\mu x + \sigma\upsilon\nu x + c$

όμως $f(0) = 1 \Leftrightarrow \eta\mu 0 + \sigma\upsilon\nu 0 + c = 1 \Leftrightarrow c = 0$

Άρα $f(x) = \eta\mu x + \sigma\upsilon\nu x$

B1. $f''(x) = (f'(x))' = (\sigma\upsilon\nu x - \eta\mu x)' = -\eta\mu x - \sigma\upsilon\nu x = -(\eta\mu x + \sigma\upsilon\nu x) = -f(x)$

B2.
$$\lim_{h\to 0} \frac{f^3\left(\frac{\pi}{6}+h\right) + \left(f''\left(\frac{\pi}{6}\right)\right)^3}{h} = \lim_{h\to 0} \frac{f^3\left(\frac{\pi}{6}+h\right) - f^3\left(\frac{\pi}{6}\right)}{h} =$$

$$= \lim_{h\to 0} \frac{\left(f\left(\frac{\pi}{6}+h\right) - f\left(\frac{\pi}{6}\right)\right)\left(f^2\left(\frac{\pi}{6}+h\right) + f\left(\frac{\pi}{6}+h\right)\cdot f\left(\frac{\pi}{6}\right) + f^2\left(\frac{\pi}{6}\right)\right)}{h} =$$

$$= f'\left(\frac{\pi}{6}\right) \cdot \lim_{h\to 0}\left(f^2\left(\frac{\pi}{6}+h\right) + f\left(\frac{\pi}{6}+h\right)\cdot f\left(\frac{\pi}{6}\right) + f^2\left(\frac{\pi}{6}\right)\right) = f'\left(\frac{\pi}{6}\right)\cdot 3f^2\left(\frac{\pi}{6}\right)$$

$$= \left(\sigma\upsilon\nu\frac{\pi}{6} - \eta\mu\frac{\pi}{6}\right)\cdot 3\left(\eta\mu\frac{\pi}{6} + \sigma\upsilon\nu\frac{\pi}{6}\right)^2 = 3\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)\cdot\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right)^2 =$$

$$= 3\cdot\frac{\sqrt{3}-1}{2}\cdot\frac{(\sqrt{3}+1)^2}{4} = \frac{3(\sqrt{3}-1)(\sqrt{3}+1)(\sqrt{3}+1)}{8} = \frac{3(\sqrt{3}^2-1)(\sqrt{3}+1)}{8} = \frac{3(\sqrt{3}+1)}{4}$$

Γ1. $f'(x) = \sigma\upsilon\nu x - \eta\mu x$, $f'(x) = 0 \Leftrightarrow \sigma\upsilon\nu x - \eta\mu x = 0 \Leftrightarrow \varepsilon\varphi x = 1 \Leftrightarrow x = \frac{\pi}{4}$

$x \in \left[0, \frac{\pi}{4}\right]$

x	$-\infty$	0		$\pi/4$	$+\infty$
f'	//////		+	//////	
f	//////		↗	//////	

$x = \frac{\pi}{6}$: $f'\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} - \frac{1}{2} > 0$

Γ2. Αφού f ↑ τότε f 1-1 οπότε αντιστρέφεται

$\Delta = f(D) = \left[f(0), f\left(\frac{\pi}{4}\right)\right] = [1, \sqrt{2}]$

$f(0) = \eta\mu 0 + \sigma\upsilon\nu 0 = 1$

$f\left(\frac{\pi}{4}\right) = \eta\mu\frac{\pi}{4} + \sigma\upsilon\nu\frac{\pi}{4} = \sqrt{2}$

Γ3. $\eta\mu\left(f^{-1}(x)\right) + \sigma\upsilon\nu\left(f^{-1}(x)\right) = x^2 \Leftrightarrow f\left(f^{-1}(x)\right) = x^2 \Leftrightarrow x = x^2 \Leftrightarrow x^2 - x = 0$

$\Leftrightarrow x(x-1) = 0 \Leftrightarrow x = 0$ ή $x = 1$

Δ. $\eta\mu x + \sigma\upsilon\nu x = x + 1 \Leftrightarrow f(x) = x + 1$

$f'(0) = \sigma\upsilon\nu 0 - \eta\mu 0 = 1$

ε: $y - f(0) = f'(0)(x - 0) \Leftrightarrow y - 1 = 1x \Leftrightarrow y = x + 1$

Άρα $f(x) = y$ οπότε $x = 0$