Θέμα 02

A1. $D_h = D_{f\circ g} = \begin{cases} x \in D_g \\ g(x) \in D_f \end{cases} = \begin{cases} x > -1 \\ \ln(x+1) \in \mathbb{R} \end{cases} = D = (-1, +\infty)$

$(f\circ g)(x) = f(g(x)) = f(\ln(x+1)) = e^{\ln(x+1)} + \ln(x+1) - 1 = x + \cancel{1} + \ln(x+1) - \cancel{1}$

Άρα $h(x) = \ln(x+1) + x \quad , \ x \in D$

A2. $h'(x) = \dfrac{1}{x+1} \cdot (x+1)' + 1 = \dfrac{1}{x+1} + 1 > 0 \quad \Rightarrow h \uparrow$

A3. $\lim\limits_{x\to 1} \dfrac{h(1)\cdot h(x) - \ln^2 2 - 2\ln 2 - 1}{x-1} = \lim\limits_{x\to 1} \dfrac{h(1)\cdot h(x) - (\ln 2 + 1)^2}{x-1} \ \underline{\underline{h(1) = \ln 2 + 1}}$

$= \lim\limits_{x\to 1} \dfrac{h(1)\cdot h(x) - h^2(1)}{x-1} = h(1)\cdot \lim\limits_{x\to 1} \dfrac{h(x) - h(1)}{x-1} = h(1)\cdot h'(1) = (\ln 2 + 1)\cdot \dfrac{3}{2}$

$= \dfrac{3\ln 2 + 3}{2} = \dfrac{\ln 8 + 3}{2}$

A4. $\ln(x+2) + x = \ln 2 \Leftrightarrow \ln(x+1+1) + x = \ln 2 \Leftrightarrow \ln((x+1)+1) + x + 1 = \ln 2 + 1$

$\Leftrightarrow h(x+1) = h(1) \overset{h\uparrow}{\Longleftrightarrow} x+1 = 1 \Leftrightarrow x = 0$

Αν $x = 0$. $\ln 2 = \ln 2$ προφανές

B1. $h \uparrow$ οπότε 1-1 άρα h αντιστρέψιμη

$D_{h^{-1}} = h(D) = \left(\lim\limits_{x\to -1} h(x), \lim\limits_{x\to +\infty} h(x)\right) = (-\infty, +\infty) = \mathbb{R}$

B2. $h^{-1}(x) = x \overset{h\uparrow}{\Longleftrightarrow} h(x) = x \Leftrightarrow \ln(x+1) + x = x \Leftrightarrow \ln(x+1) = 0 \Leftrightarrow x+1 = 1 \Leftrightarrow x = 0$

B3.